AMERICAN UNIVERSITY OF BEIRUT
Department of Electrical and Computer Engineering
EECE340 Signals and Systems -Summer 2011

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Quiz 1, July 13, 2011

Directions:

- Write down your name *in ink* below and your initials on all the pages. DO IT NOW!
- You have 1.5 hours to complete the quiz.
- Enter ALL your work and your answers on the answer booklet. You can use the back of these pages for scratch. I will ONLY grade the work you neatly transfer to the booklet.
- Answers must be explained or derived. DO NOT just write down an answer, unless otherwise indicated.
- It is a good idea to read the whole test before you begin. Problems are divided into several parts with percentages indicated. You might be able to solve different parts independently.
- DO NOT talk to any of your colleagues under any circumstances. You will be penalized without warning.

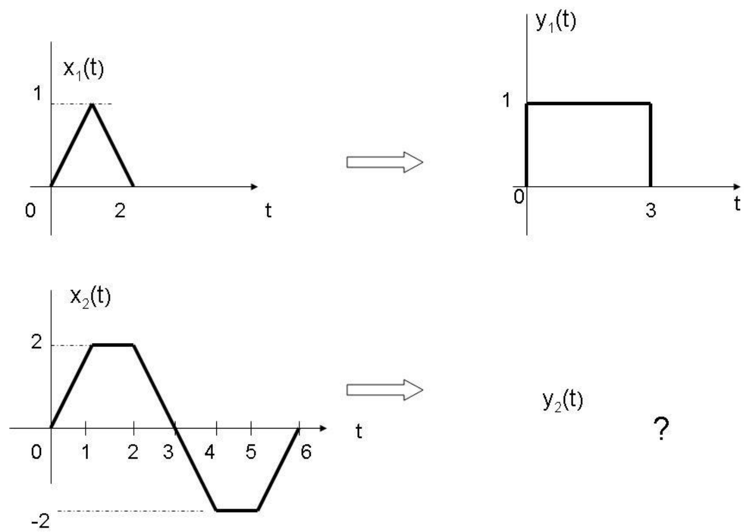
YOUR NAME HERE:

PROBLEM 1 (50%)
DIFFERENT PARTS OF THIS PROBLEM ARE INDEPENDENT

a) Determine whether the following DT system with input $x[n]$ and output $y[n]$ is (i) Linear and (ii) Time invariant. Prove or give a counter example.

$$y[n] = x[n-1] + |nx[n]|$$

b) A CT LTI system was tested with the input $x_1[n]$ shown below to give the output $y_1[n]$ as shown in figure below. Determine the output $y_2[n]$ if the input $x_2[n]$ is as shown in the figure.



c) Consider the following Linear difference equation describing the dynamics of a DT Causal LTI system with input $x[n]$ and output $y[n]$

$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 4x[n] + \frac{1}{2}x[n-2]$$

c-i Find the Zero State response of this system to an input $x[n] = \left(\frac{1}{2}\right)^n u[n]$

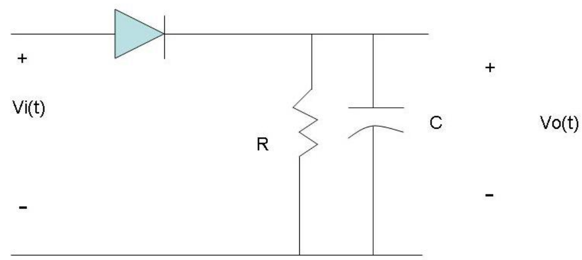
c-ii Find the unit sample response $h[n]$ of this system. Is it stable? explain.

d) The transfer function of CT causal LTI system is unknown. Experiments show that:

- If the input is a unit step $u(t)$, the output is given by $y(t) = \alpha e^{-4t} + \beta e^{-5t}$, $t \geq 0$
- If the input is $x(t) = e^{-t}u(t)$, the output is $y(t) = 2e^{-t} + \gamma e^{-4t} + \theta e^{-5t}$, $t \geq 0$

Do you have sufficient information to determine the transfer function $H(s)$? If so, find $H(s)$.

e) Determine whether the following circuit with input $V_i(t)$ and output $V_o(t)$ is (i) Linear and (ii) Time invariant. Prove or give a counter example.



f) An LTI system is described by the following system function:

$$\frac{Y(s)}{X(s)} = H(s) = \frac{s+2}{s^2+s}$$

(f-1)- What are the poles of $H(s)$? Is the system stable?

(f-2)- Find the Zero State Response $y_{SZR}(t)$ of the corresponding system for an input $x(t) = 4e^{-t}u(t)$.

(f-3)- Find the total output of the system $y(t)$, $t > 0$ for the input given above $x(t) = 4e^{-t}u(t)$, knowing that $y(0) = 0$, $\dot{y}(0) = 0$.

PROBLEM 2 (15%)

A CT causal LTI system is described by the following linear ordinary differential equation

$$\frac{d^3y(t)}{dt^3} + 7\frac{d^2y(t)}{dt^2} + 14\frac{dy(t)}{dt} + 8y(t) = x + \frac{dx(t)}{dt}$$

a) Find a state space realization of this system *in matrix form.*

b) If an input $x(t) = \cos(t)u(t)$ is applied for a long time and the response is observed on an oscilloscope, sketch what you would observe. Please determine any magnitude values and phase shifts in the output.

PROBLEM 3 (15%)

An overweight Homer Simpson is to perform a daily exercise to control his weight while limiting his daily food intake. A new fat-analysis machine provided a careful assessment of his weight dynamics and showed that:

On a given day,

- Homer's weight $w[n]$ is proportional to his weight on the previous day $w[n-1]$
- Homer's weight is reduced by a factor proportional to his weight and the number of swimming laps he completes two days ago.
- Homer's weight is increased by a factor of 1/250 of his calories intake on that day.

To make it easier, the equation of his weight dynamics is thus given by

$$w[n+2] = w[n+1] - \frac{N}{50}w[n] + \frac{C}{250}$$

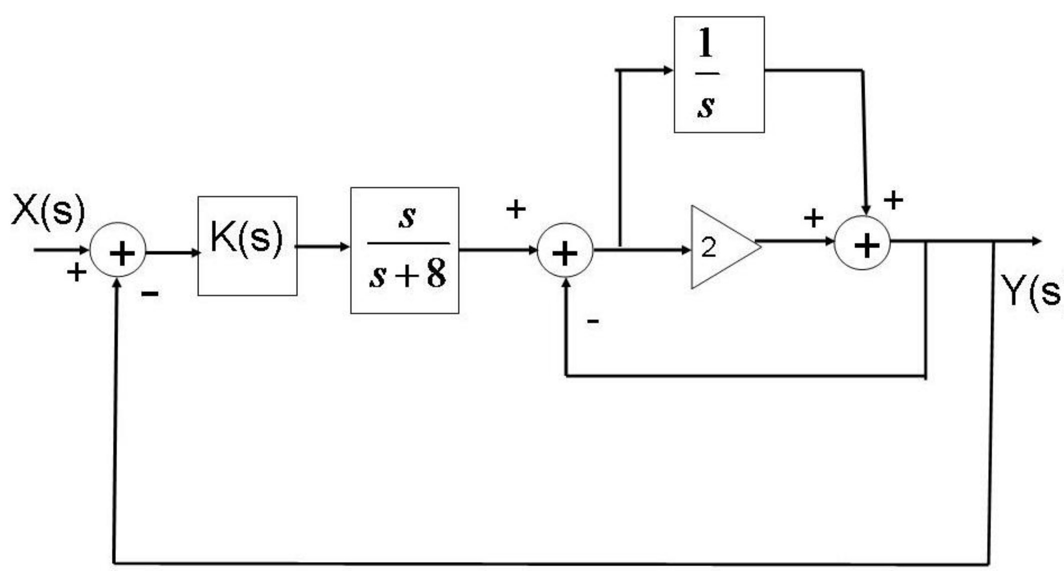
Where N is the number of laps completed on any day and C is the daily calories intake.

a) If Mr Simpson's daily food intake provides a $C = 1500$ Calories, and ignoring his initial weight, determine the constant number of laps he is to complete such that his final weight is $\lim_{n\to\infty} w[n] = 75$ Kilograms

b) If he keeps to a daily intake of $C = 3500$ calories in fatty foods, can Homer achieve his target weight? explain.

PROBLEM 4 (20%)

Consider the CT causal LTI system shown in figure below.



a) If the controller $K(s)$ is a simple proportional control $K(s) = K_o$, find the range of gain K_o for which the overall system is stable.

b) The system is now required to track unit step inputs $u(t)$. Determine which, if any, of the following controller can achieve tracking?

(i)- $K(s) = K_o$

(ii)- $K(s) = K_o + K_1 s$

(iii)- $K(s) = K_o + \dfrac{K_1}{s^2}$